NAME. Marking Scheme ....INDEX NO..............ADM NO............

CANDIDATE'S SIGN..........................DATE..

SCHOOL...

LANJET JOINT EXAMINATIONS

Kenya Certificate of Secondary Education (K.C.S.E)

121/2
MATHEMATICS
PAPER 2
DECEMBER 2021
TIME: 2 ½ HOURS

INSTRUCTIONS TO CANDIDATES

1) Write **your name** and **index number** in the spaces provided above.
2) Sign and write the date of examination in the spaces provided above.
3) This paper consists of two section **I** and **II**.
4) Answer **ALL** questions in section **I** and only **five** questions from section **II**.
5) Answers and working must be written on the question paper in the spaces provided below each question.
6) Marks may be given for correct working even if the answer is wrong.
7) Non-programmable electronic calculators may be used.

FOR EXAMINERS' USE ONLY.

SECTION I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

SECTION II

17	18	19	20	21	22	23	24	TOTAL

Grand Total

1. Using logarithm tables evaluate. (4mks)

$$\sqrt{\frac{\log 32.36 \times 0.1233^2}{\sin 8.91^0}}$$

log32	1.510×10^0	$0.1790 \longrightarrow 0.1790$ +
0.1233	1.233×10^{-1}	$\bar{1}.0910 \times 2 = \bar{2}.1820$
		$\bar{2}.3610$ −
$\sin 8.91^\circ \longrightarrow$		$\bar{1}.1900 \longrightarrow \bar{1}.1900$ −
		$\bar{2}.$ $\quad \frac{\bar{1}.1710}{2} = \bar{1}.5855$
$0.385 =$	3.850×10^{-1}	$\longleftarrow \bar{1}.5855$

2. A circle which passes through the point (-1, 7) has its center at (3, -3). Determine the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. Where a, b and c are constants. (3 mks)

$r = \begin{pmatrix}3\\-3\end{pmatrix} - \begin{pmatrix}-1\\7\end{pmatrix} = \begin{pmatrix}4\\-10\end{pmatrix}$

$r = \sqrt{16 + 100} = \sqrt{116}$

$r^2 = 116$

$(x-3)^2 + (y+3)^2 = 116$

$x^2 - 6x + 9 + y^2 + 6y + 9 - 116 = 0$

$x^2 + y^2 - 6x + 6y - 98 = 0$

3. Expand $(1 + 2x)^7$ up to the term in x^3. Use your expansion to estimate $(1.02)^7$ to 4dp (3mks)

$1\,1^7(2x)^0 + 7\,1^6(2x)^1 + 21\,1^5(2x)^2 + 35\,1^4(2x)^3$

$1 + 14x + 84x^2 + 280x^3$

$2x = 0.02$

$x = 0.01$

$1 + 14(0.01) + 84(0.01)^2 + 280(0.01)^3$

$= 1.1487$

4. Solve $1 + 2 \sin 2x = 0$ for $0^0 \leq 360^0$. (3 mks)

$2\sin 2x = -1$

$\sin 2x = -\frac{1}{2}$

$2x = 210^\circ, 330^\circ, 570^\circ, 690^\circ$

$x = 105^\circ, 165^\circ, 285^\circ, 345^\circ$

$x = 105^\circ, 165^\circ, 285^\circ, 345^\circ$

5. Given that x = 2i +j – 2k, y =-3i + 4j – k and z = -5i + 3j + 2k and that p = 3x – y + 2z, Calculate the magnitude of p correct to 3 significant figures. (3 mks)

$$P = 3\begin{pmatrix}2\\1\\-2\end{pmatrix} - \begin{pmatrix}-3\\4\\-1\end{pmatrix} + 2\begin{pmatrix}-5\\3\\2\end{pmatrix}$$

$$\begin{pmatrix}6\\3\\-6\end{pmatrix} - \begin{pmatrix}-3\\4\\-1\end{pmatrix} + \begin{pmatrix}-10\\6\\4\end{pmatrix}$$

$$P = \begin{pmatrix}-1\\5\\-1\end{pmatrix}$$

$$|P| = \sqrt{(-1)^2+(5)^2+(-1)^2}$$

$$= 5.20 \text{ units.}$$

6. Calculate the semi interquartile range for the following set of data. (4 mks)

16, 42, 41, 6, 20, 28, 19, 23, 15

6, 15, 16, 19, 20, 23, 28, 41, 42

$$Q_1 = \frac{15+16}{2} = 15.5$$

$$Q_3 = \frac{28+41}{2} = 34.5$$

$$\frac{34.5 - 15.5}{2} = 9.5$$

7. Make h the subject of the formula (3 mks)

$$\frac{E}{X} = \sqrt{\frac{h - 0.5}{I - h}}$$

$$\frac{E^2}{X^2} = \frac{h - 0.5}{1 - h}$$

$$E^2 - E^2h = X^2h - 0.5X^2$$

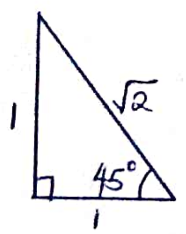
$$E^2 + 0.5X^2 = X^2h + E^2h$$

$$h(X^2 + E^2) = E^2 + 0.5X^2$$

$$h = \frac{E^2 + 0.5X^2}{X^2 + E^2}$$

8. Evaluate by rationalizing the denominator and leaving your answer in surd form. (3 mks)

$$\frac{\sqrt{8}}{1 + \text{Cos}45^0}$$



$$\text{Cos}45° = \frac{1}{\sqrt{2}}$$

$$\frac{2\sqrt{2}\left(1 - \frac{1}{\sqrt{2}}\right)}{\left(1 + \frac{1}{\sqrt{2}}\right)\left(1 - \frac{1}{\sqrt{2}}\right)}$$

$$\frac{2\sqrt{2} - 2}{1 - \frac{1}{2}}$$

$$\frac{2\sqrt{2} - 2}{\frac{1}{2}}$$

$$4\sqrt{2} - 4$$

9. A dealer has two types of grades of tea, A and B. Grade A costs sh140 per kg. Grade B costs sh. 160 per kg. If the dealer mixes A and B in the ratio 3:5 to make a brand of tea which he sells at sh. 180 per kg, calculate the percentage profit that he makes. (3 mks)

$\frac{(140 \times 3) + (160 \times 5)}{8} = sh152.50$

$180 - 152.5 = sh27.50$

$\frac{27.50}{152.5} \times 100\% = 18.03\%$

10. Onyango bought a refrigerator whose cash price is Sh.84,000 on hire purchase. He made a cash deposit of sh. 20,000 and the 15 monthly instalments of Sh. 6,000. Calculate the rate of interest per month. (3 mks)

$A = 15 \times 6000 = 90000$

$P = 84000 - 20,000 = 64,000$

$90,000 = 64000(1 + r/100)^{15}$

$1.40625 = (1 + r/100)^{15}$

$1 + r/100 = 1.02299$

$r = 0.02299 \times 100$

$r = 2.299\%$

11. Estimate the area bounded by the curve $y = \frac{1}{2}x^2 + 1$, $x = 0$, $x = 3$ and the x-axis using the mid-ordinate rule. Use three strips (3 mks)

$h = \frac{3-0}{3} = 1$

X	0.5	1.5	2.5
Y	1.125	2.125	4.125

$A = h(y_1 + y_2 + y_3)$

$= 1(1.125 + 2.125 + 4.125) = 7.375$ square units

12. A variable y varies as the square of x and inversely as the square root of z. Find the percentage change in y when x is increased by 5% and z reduced by 19%. (3mks)

$y = \frac{kx^2}{\sqrt{z}}$

$y_n = \frac{k(1.05x)^2}{\sqrt{0.81z}}$

$y_n = \frac{1.225xk}{\sqrt{z}}$

Change in $y = \frac{1.225kx^2}{\sqrt{z}} - \frac{kx^2}{\sqrt{z}} = \frac{0.225kx^2}{\sqrt{z}}$

$\frac{\frac{0.225kx^2}{\sqrt{z}}}{\frac{kx^2}{\sqrt{x}}} \times 100\%$

$= 22.5\%$ increase

13. A two digit number is such that the sum of ones and tens digits is ten. If the digits are reversed, the new number formed exceeds the original number by 54. Find the original number. (3mks)

$x + y = 10$

$XY = 28$

$yx - xy = 54$

$9y - 9x = 54$

$y - x = 6$

$\underline{y + x = 10}$

$-2x = -4$

$x = 2$

$y = 8$

14. The current price of a vehicle is Sh500 000. If the vehicle depreciates at a rate of 15% p.a., find the number of year it will take for its value to fall to Sh180 000. (3mks)

$80{,}000 = 500000\left(1 - \frac{15}{100}\right)^n$

$0.36 = 0.85^n$

$\log 0.36 = n \log 0.85$

$n = \frac{\log 0.36}{0.85} = 6.286$ yrs.

15. The gradient function of a curve at any point (x, y) is given as $6x^2$. Given that the curve passes through the point (1, 5), find its equation. (3mks)

$\frac{dy}{dx} = 6x^2$

$y = 2x^3 + c$

$5 = 2(1)^3 + c$

$c = 3$

$y = 2x^3 + 3$

16. Vector **m** passes through the points (6, 8) and (2, 4). Vector **n** passes through (x, -2) and (-5, 0), if **m** is parallel to **n** determine the value of x. (3mks)

$M = \begin{pmatrix}6\\8\end{pmatrix} - \begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}4\\4\end{pmatrix}$

$n = \begin{pmatrix}-5\\0\end{pmatrix} - \begin{pmatrix}x\\-2\end{pmatrix} = \begin{pmatrix}-5-x\\2\end{pmatrix}$

$n = km$

$4\cancel{km} = 2$

$4m = 2$

$m = \frac{1}{2}$

$\frac{1}{2}(4) = -5 - x$

$2 = -5 - x$

$7 = -x$

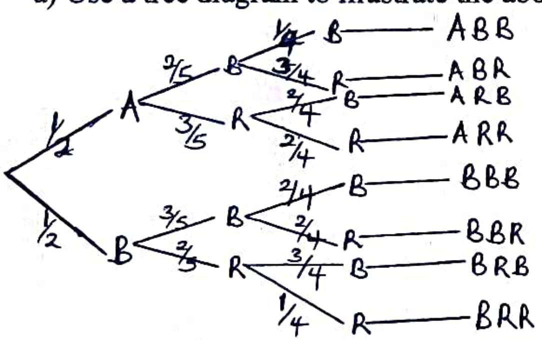
$x = -7$

SECTION II

*Answer any **FIVE** questions from this section*

17. Two bags A and B contain red and blue balls. Bag A contains 2 blue balls and 3 red balls while bag B contains 3 blue balls and 2 red balls. A bag is selected at random and two balls picked without replacement.

a) Use a tree diagram to illustrate the above information (2mks)



b) Use the tree diagram above to find the probability of picking

i) Balls of the same colour (2mks)

P(A BB) or P(A RR) or P(BBB) or P(BRR)

$\left(\frac{1}{2}\times\frac{2}{5}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{3}{5}\times\frac{2}{4}\right)+\left(\frac{1}{2}\times\frac{3}{5}\times\frac{2}{4}\right)+\left(\frac{1}{2}\times\frac{3}{5}\times\frac{1}{4}\right)$

$\frac{1}{20}+\frac{3}{20}+\frac{3}{20}+\frac{1}{20}=\frac{2}{5}$

ii) A red ball followed by a blue ball (2mks)

P(ARB) or P(BRB)

$\left(\frac{1}{2}\times\frac{3}{5}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{2}{5}\times\frac{3}{4}\right)$

$\frac{3}{20}+\frac{3}{20}=\frac{3}{10}$

iii) At least a blue ball (2mks)

$1-[P(ARR) \text{ or } P(BRR)]$

$1-\left[\left(\frac{1}{2}\times\frac{2}{5}\times\frac{1}{4}\right)+\left(\frac{1}{2}\times\frac{3}{5}\times\frac{2}{4}\right)\right]$

$1-\frac{1}{5}=\frac{4}{5}=\frac{4}{5}$

iv) One red ball (2mks)

P(ABR) or P(ARB) or P(BRB) or P(BBR)

$\left(\frac{1}{2}\times\frac{2}{5}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{3}{5}\times\frac{2}{4}\right)+\left(\frac{1}{2}\times\frac{2}{5}\times\frac{3}{4}\right)+\left(\frac{1}{2}\times\frac{3}{5}\times\frac{2}{4}\right)$

$\frac{6}{40}+\frac{6}{40}+\frac{6}{40}+\frac{6}{40}$

$=\frac{24}{40}=\frac{3}{5}$

18. The 2nd and the 5th terms of an arithmetic progression are 8 and 17 respectively, the 2nd, 10th and the 42nd terms of the A.P form the first three terms of a geometric progression. Find.

a) The 1st term and the common difference of the A.P. (3mks)

$$a+d=8$$
$$a+4d=17$$
$$-3d=-9$$
$$d=3$$
$$a+3=8$$
$$a=5$$
$$d=3$$

b) The first three terms of G.P and the 10th term of the G.P. (4mks)

$2^{nd} = 5+3 = 8$

$10^{th} = 5+27 = 32$

$42^{nd} = 5+123 = 128$

$\therefore$ G.P. is $8, 32, 128$

$a=8$

$r = \frac{32}{8} = 4$

$T_{10} = ar^{n-1}$

$T_{10} = 8(4)^9$

$= 2097152$

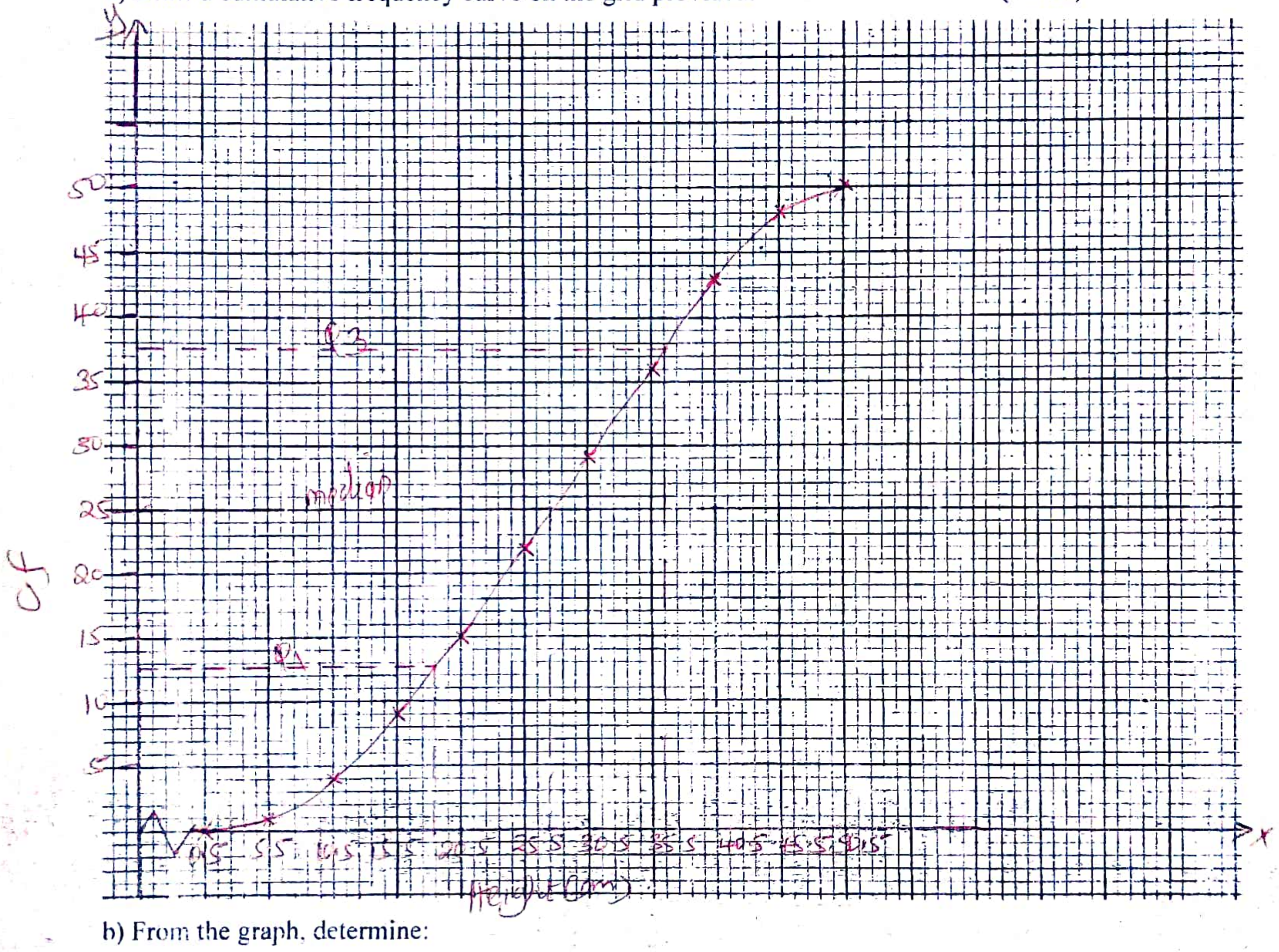
c) The sum of the first 10 terms of the G.P. (3mks)

$$S_n = \frac{a(r^n-1)}{r-1}$$
$$= \frac{8(4^{10}-1)}{4-1}$$
$$= \frac{8}{3}(1048575)$$
$$= 2,796,200$$

19.

Height in cm	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
Number of plants	1	3	5	6	7	7	7	7	5	2
cf	1	4	9	15	22	29	36	43	48	50

a) Draw a cumulative frequency curve on the grid provided. (4 mks)



b) From the graph, determine:

i) The median height (2mks)

27.5 cm

ii) The quartile deviation (2mks)

$q_1 = 18.5$

$q_3 = 36.5$

$\frac{36.5 - 18.5}{2} = \frac{18}{2} = 9$

iii) The range of the height of the middle 50% of the plants. (2 mks)

18.5 to 36.5
OR
18.5 – 36.5

18

20(a). Complete the table below for the function $y = 3x^2 - 2x - 1$ for $-3 \leq x \leq 4$. (3 marks)

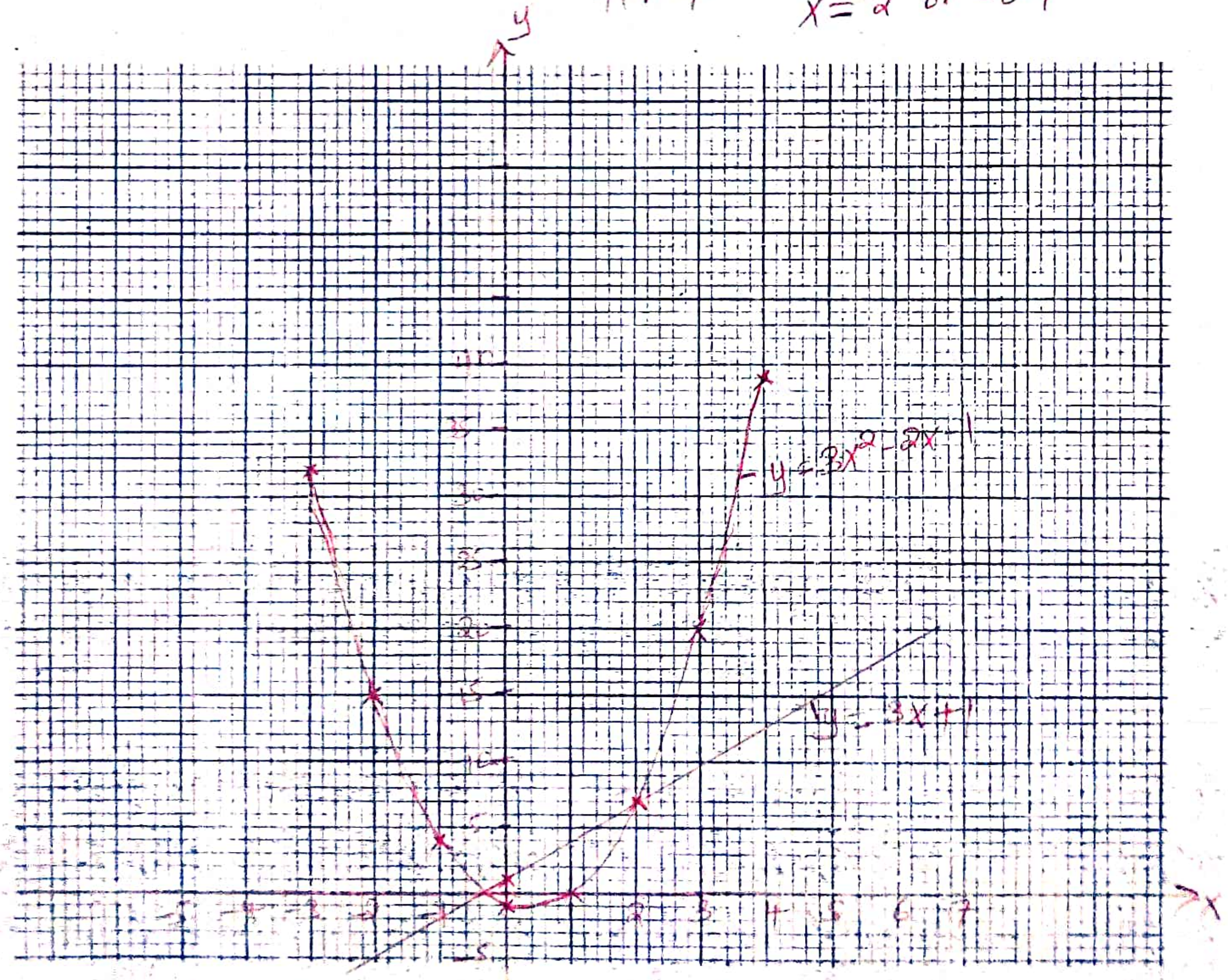
X	-3	-2	-1	0	1	2	3	4
$3x^2$	27	12	3	0	3	12	27	48
-2x	6	4	2	0	-2	-4	-6	-8
-1	-1	-1	-1	-1	-1	-1	-1	-1
Y	32	15	4	-1	0	7	20	39

(b) Draw the graph of $y = 3x^2 - 2x - 1$. (4 marks)

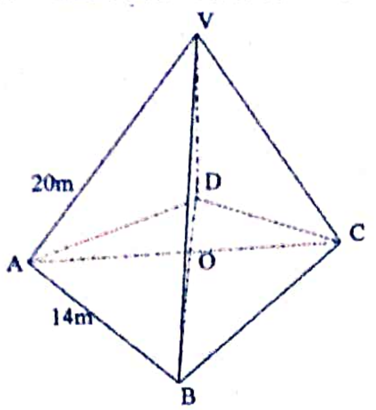
(c) Draw the line $y = 3x + 1$ on the same axis hence find the values of x for which $y = 3x + 1$ and $y = 3x^2 - 2x - 1$ are equal. (3 marks)

x	0	2
y	1	7

$x = 2$ or -0.4



21. The diagram below shows a right pyramid with a square base ABCD and vertex V. O is the centre of the base. AB = 14m, VA = 20m and N is the midpoint of BC.



Find;

a) The lengths of BO, VO and VN (3 mks)

$BO = \frac{\sqrt{14^2+14^2}}{2} = \frac{19.799cm}{2} = 9.899cm$

$VO = \sqrt{20^2 - 9.899^2} = 17.38\ cm$

$VN = \sqrt{20^2 - 7^2} = 18.73cm$

b) The angle between VO and plane VBC (3 mks)

$\sin\theta = \frac{7}{18.73} = 0.3737$

$\theta = \sin^{-1}0.3737$

$= 21.94^\circ$

c) The angle between VB and base ABCD (2 mks)

$\tan\theta = \frac{17.38}{9.899}$

$\theta = \tan^{-1}1.756$

$\theta = 60.34^\circ$

d) The volume of the pyramid VABCD (2 mks)

$\frac{1}{3} \times 14 \times 14 \times 17.38 = 1135.49cm^3$

22. A transformation represented by matrix $\begin{bmatrix}2 & 1\\1 & -2\end{bmatrix}$ maps A(0,0), B(2,0), C(2,3) and D(0,3) onto $A^1B^1C^1$ and D^1 respectively.

a) Draw ABCD and its image $A^1B^1C^1D^1$ (4 marks)

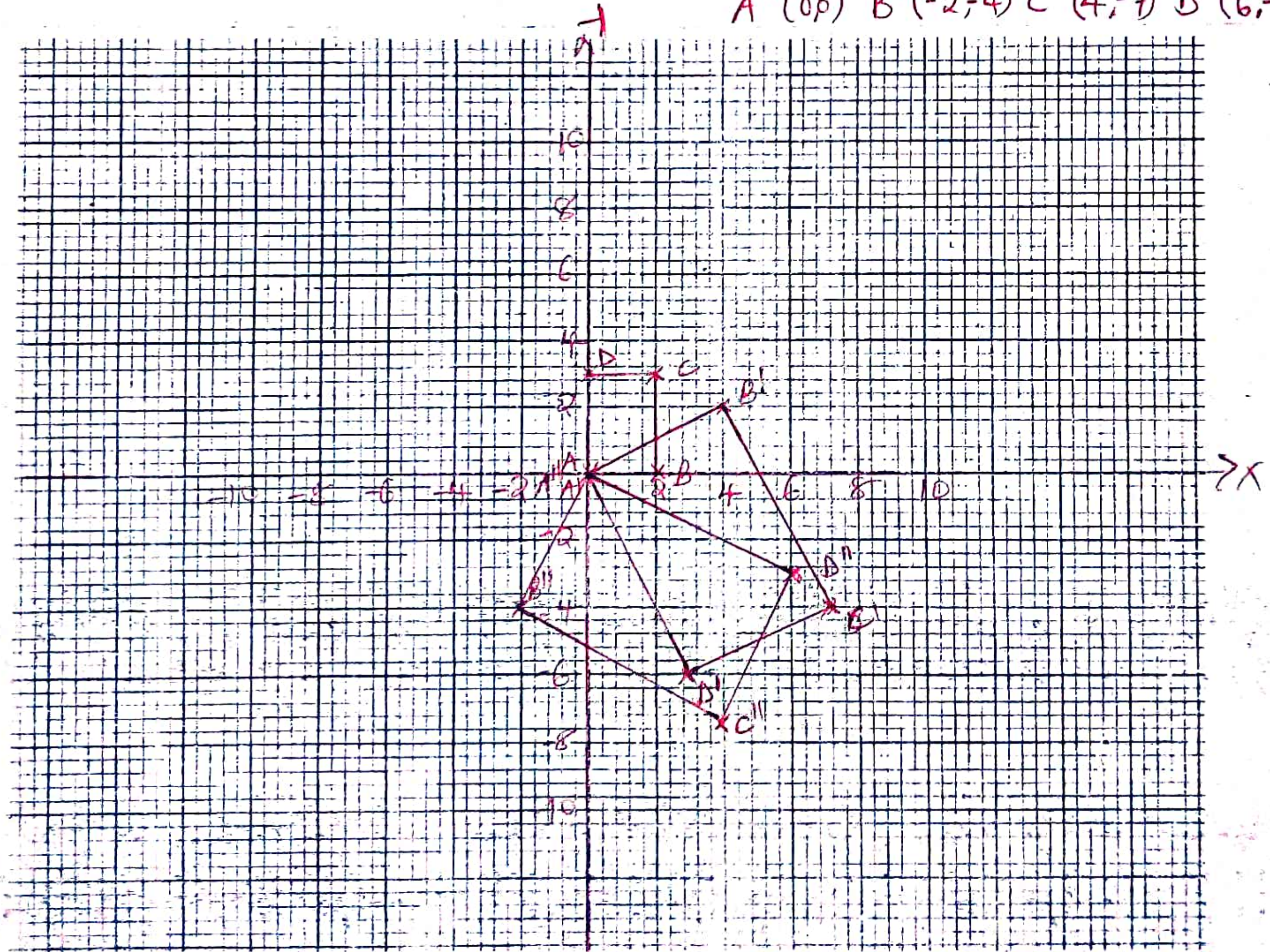
$$\begin{bmatrix}2 & 1\\1 & -2\end{bmatrix}\begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\\begin{bmatrix}0 & 2 & 2 & 0\\0 & 0 & 3 & 3\end{bmatrix}\end{array} = \begin{array}{c}\begin{array}{cccc}A' & B' & C' & D'\end{array}\\\begin{bmatrix}0 & 4 & 7 & 3\\0 & 2 & -4 & -6\end{bmatrix}\end{array}$$

$A'(0,0)$ $B'(4,2)$ $C'(7-4)$ $D'(3,-6)$

b) A transformation represented by $\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$ maps $A^1B^1C^1D^1$ on $A^{11}B^{11}C^{11}D^{11}$. Plot $A^{11}B^{11}C^{11}D^{11}$ on the same graph. (3 marks)

$$\begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}\begin{array}{c}\begin{array}{cccc}A' & B' & C' & D'\end{array}\\\begin{bmatrix}0 & 4 & 7 & 3\\0 & 2 & -4 & -6\end{bmatrix}\end{array} = \begin{array}{c}\begin{array}{cccc}A'' & B'' & C'' & D''\end{array}\\\begin{bmatrix}0 & -2 & 4 & 6\\0 & -4 & -7 & -3\end{bmatrix}\end{array}$$

$A''(0,0)$ $B''(-2,-4)$ $C''(4,-7)$ $D''(6,-3)$



c) Determine the matrix of a single transformation that maps $A^{11}B^{11}C^{11}D^{11}$ onto ABCD (3 marks)

$$\begin{bmatrix}0 & 2 & 2 & 0\\0 & 0 & 3 & 3\end{bmatrix} = \begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}0 & -2 & 4 & 6\\0 & -4 & -7 & -3\end{bmatrix}$$

$$\begin{bmatrix}0 & 2 & 2 & 0\\0 & 0 & 3 & 3\end{bmatrix} = \begin{bmatrix}0 & -2a-4b & 4a-7b & 6a-3b\\0 & -2c-4d & 4c-7d & 6c-3d\end{bmatrix}$$

$-2a-4b=2$
$6a-3b=0$
$3b=6a$
$b=2a$
$-2a-4(2a)=2$

$-2a-8a=2$
$-10a=2$
$a=-0.2$
$b=2(-0.2)$
$=-0.4$

$-2c-4d=0$; $4d=-2c$
$c=-2d$
$4c-7d=3$
$4(-2d)-7d=3$
$-15d=3$; $d=-0.2$
$c=-0.4$

$T=\begin{bmatrix}-0.2 & -0.4\\-0.4 & -0.2\end{bmatrix}$

23. a) The acceleration of a particle t seconds after passing a fixed point P is given by $a = 3t - 3$. Given that the velocity of the particle when $t = 2$ is 5 m/s, find;

i) Its velocity when $t = 4$ seconds (3 marks)

$$V = \frac{3t^{1+1}}{2} + \frac{3t^{0+1}}{1} + c$$

$$V = \tfrac{3}{2}t^2 - 3t + c$$

$$5 = \tfrac{3}{2}(2)^2 - 3(2) + c$$

$$5 = c$$

$$V = \tfrac{3}{2}t^2 - 3t + 5$$

$$V = \tfrac{3}{2}(4)^2 - 3(4) + 5$$

$$24 - 12 + 5$$

$$= 17\text{m/s}$$

ii) Its displacement at this time (3 marks)

$$S = \frac{\tfrac{3}{2}t^{2+1}}{3} - \frac{3t^2}{2} + \frac{5t^{0+1}}{1} + c$$

$$S = \tfrac{1}{2}t^3 - \tfrac{3}{2}t^2 + 5t^1 + c$$

$S = 0, t = 0, C = 0$

$$S = \tfrac{1}{2}t^3 - \tfrac{3}{2}t^2 + 5t$$

$$S = \tfrac{1}{2}(4)^3 - \tfrac{3}{2}(4)^2 + 5(4)$$

$$32 - 24 + 20$$

$$= 28\text{m}$$

(b) Find the exact area bounded by the graph $x = 9y - y^3$ and the y-axis (4 marks)

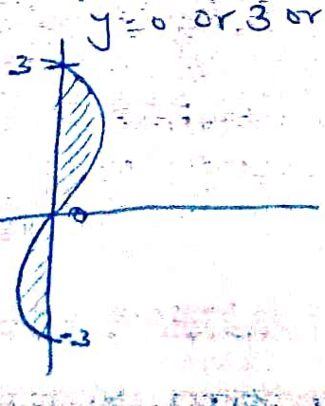
$$\int \left(\frac{9y^{1+1}}{2} - \frac{y^{3+1}}{4}\right) dy$$

y-intercept → $9y - y^3 = 0$

$$y(9 - y^2) = 0$$

$$y = 0 \text{ or } (3-y)(3+y) = 0$$

$$y = 0 \text{ or } 3 \text{ or } -3$$



$$\left[\tfrac{9}{2}y^2 - \frac{y^4}{4}\right]_0^3$$

$$\left(\tfrac{9}{2}(3)^2 - \frac{81}{4}\right) - (0)$$

$$\frac{81}{2} - \frac{81}{4} = 20.25$$

$$\left[\tfrac{9}{2}y^2 - \frac{y^4}{4}\right]_{-3}^0$$

$$0 - \left(\frac{81}{2} - \frac{81}{4}\right) = -20.25$$

Total area = 20.25 + 20.25

= 50.5 square units

24. The position of 3 cities P, Q and R are (15⁰N, 20⁰W) (50⁰N, 20⁰W) and (50⁰N, 60⁰E) respectively.

a) Find the distance in nautical miles between:

(i) Cities P and Q (2 marks)

$\theta = 50 - 15 = 35^\circ$

$35 \times 60 = 2100\text{nm}$

(ii) Cities P and R, via city Q (3 marks)

$\theta = 60 + 20 = 80^\circ$

$80 \times 60 \cos 50^\circ = 3085.38\text{nm}$

$3085.38 + 2100\text{nm}$

$= 5185.38\text{nm}$

b) A plane left city P at 0250h and flew to city Q where it stopped for 3 hours then flew on to city R, maintaining a ground speed of 900 knots throughout. Find:

(i) The local time at city R when the plane left city P (3 marks)

$\theta = 60 + 20 = 80^\circ$

$1^\circ = 4\text{mins}$

$80^\circ = X$

$X = \frac{80 \times 4}{1} = \frac{320\text{mins}}{60}$

5h 20mins

0250
520
0810

0810 hrs

(ii) The local time (to the nearest minute) at city R when the plane landed at R. (2 marks)

$t = \frac{5185.38}{900} = 5\text{hrs } 46\text{mins}$

$+ 3\text{hrs } 00\text{mins}$

$8\text{hrs } 46\text{mins}$

0810
846
1656

1656hrs.